Exam 25.2.2013  (6 assignments / 2 pages)

Since there may be English speaking students in the exam, the questions below are in English—just like in all previous exercises. Though, answers can be given in Finnish, of course.

The so-called MAOL background information and calculators are allowed in the exam.

Assignment 1.  (3 p)
Define soft matter and give everyday examples of soft matter systems. Discuss order in soft matter systems.

Assignment 2.  (5 p)
   a) Discuss the relevant time- and length scales in soft matter. Give (calculate!) a reasonable estimate for the scale (order of magnitude) of forces on cellular level. (2 p)
   b) Consider a bacterium in water environment. There is an alcohol molecule 0.1 mm away from the bacterium. Estimate the average time the bacterium will have to wait to get the alcohol, if it just sits still and lets diffusion take care of the transport. The diffusion constant of alcohol in water is $D = 10^{-5}$ cm$^2$/s. (2 p)
   c) Langevin equation is a tool in statistical physics that may be used to describe various phenomena, depending on how the terms are interpreted. Write down the Langevin equation for Brownian motion and explain what the different terms mean. (1 p)

Assignment 3.  (4 p)
Explain the following concepts and terms.
   a) Protein
   b) Mean field theory
   c) Free energy
   c) Hydrogen bond

Assignment 4.  (6 p)
Self-avoiding random walks. One means to characterize the size of a polymer is the end-to-end distance $R_{ee}$. Let us consider a linear polymer composed of $N$ monomers. Then one can define

$$R_{ee}^2 \equiv \langle [\vec{r}_N - \vec{r}_1]^2 \rangle,$$

where the brackets $\langle \rangle$ denote an average over many independent samples and $\vec{r}_1$ ($\vec{r}_N$) is the position of the first (last) monomer.

To study this quantity in the case of a self-avoiding random walk (SAW) in a plane (in two dimensions), let us consider a walk in a lattice. Let the first monomer ($N = 1$) be at the origin ($\vec{r}_1 = (0, 0)$). Then the second monomer can reside at one of the positions $(0, 1), (-1, 0), (0, -1), \text{ or} (1, 0)$. Go through all possible combinations (configurations) of the chain up to chain length $N = 3$ and determine $R_{ee}^2$ for every chain length $N = 1, 2, \text{ and } 3$.

Using the obtained results, plot the average value of $R_{ee}^2$ as a function of $N$. In particular, make this plot in a log-log scale such that you can estimate the exponent $x$ using the scaling law

$$R_{ee}^2 \sim (N - 1)^{2x}.$$

Compare your result to the experimental value for real chains $x = 0.75$ in good solvent. What is meant by good, bad, and theta solvents?
Assignment 5. (6 p)
You are at a farm and have a bucket of fresh milk at room temperature (300K). To simplify things, think about the milk as a colloidal dispersion of small spherical fat droplets (density of fat is 910 kg/m³) in water. Let’s estimate the radius of the fat droplets to be around 1 mm. You are planning to bake in the evening and need cream for that. Your plan is to wait until the fat droplets travel to the surface of the milk so you could just scoop them out. If you have a couple of hours time before you need the cream, do you think waiting is a reasonable tactic or do you need more sophisticated separation methods? Justify your answer with calculations.

On the following day you are sick of baking and just want to store all milk you got. For this you’d like process the milk so that the fat would not cream to the surface. For this you develop a super-efficient homogenizing process, which reduces the size of the fat droplets. Once you get a bucket full of homogenized milk, you let it rest until it has reached equilibrium (you do not have to care how long this takes). Next you measure the concentration (in equilibrium) of the fat droplets on the top of the bucket and on the bottom. The concentration on the bottom is 37% of that on the top. Calculate the size of the fat droplets.

Assignment 6. (6 p)
Answer either to part (a) or (b).

(a) In liquid crystals, lipid membranes, polymer systems etc. one faces the order parameter

\[ S_2 = \left( \frac{3 \cos^2 \theta - 1}{2} \right) \]

The use of order parameters such as the above one is based on Legendre polynomials. The parameters associated with these polynomials can be measured experimentally. The 2nd order polynomial used above is one natural choice for this purpose, but obviously other choices are equally possible. An alternative idea is to use, for example,

\[ p_4(\theta) = A \cos^4 \theta + B \cos^2 \theta + C, \]

when the order parameter becomes \( S_4 = \langle p_4(\theta) \rangle \).

Write down the conditions set upon an order parameter and derive the parameters of \( S_4 \) (that is, the values of \( A, B \) and \( C \), there may be more than one correct answer) so that these conditions are satisfied. Assume that the molecules are in three-dimensional space and \( A, B, C \neq 0 \).

(b) Discuss the following themes. Use drawings and/or essay-like descriptions to clarify the issue, when needed.

(b1) **Hydrophobic effect** Explain how it works and how it affects the self-assembly of lipids.

(b2) **Reynolds number** Define the Reynolds number \( Re \) and tell what the two different limits mean: \( Re \) is much smaller than one, and \( Re \) is much larger than one. If you considered the motion of cells, what value of \( Re \) would be realistic?

(b3) **Colloids** What are colloids? Discuss ways and reasons to stabilize colloidal solutions.
These might be useful:
\[
\begin{align*}
\sin 2x &= 2 \sin x \cos x \\
\sin 3x &= 3 \sin x - 4 \sin^3 x \\
\cos 2x &= 1 - 2 \sin^2 x \\
\cos 3x &= 4 \cos^3 x - 3 \cos x
\end{align*}
\]

About spherical coordinates
\[
\begin{align*}
x &= r \sin \theta \cos \phi \\
y &= r \sin \theta \sin \phi \\
z &= r \cos \theta
\end{align*}
\]

Surface element spanning from $\theta$ to $d\theta$ and $\phi$ to $d\phi$ on a spherical surface at (constant) radius $r$:
\[
dS_r = r^2 \sin \theta d\theta d\phi
\]
and the differential solid angle: $d\Omega = dS_r / r^2 = \sin \theta d\theta d\phi$

Taylor expansion
\[
\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n
\]

Boltzmann constant $1.380662 \cdot 10^{-23}$ J/K
1 amu = $1.66 \times 10^{-27}$ kg, (atomic mass unit)
viscosity of water $\eta = 10^{-3}$ kg/ms

$S_2 = \left( \frac{3 \cos^2 \theta - 1}{2} \right)$

$S = k_B \ln \Omega$

$F(r) = \frac{Q_1 Q_2}{4 \pi \epsilon_0 e^2 r^2}$

$U(r) = -\frac{A}{r} + \frac{B}{r^2}$

$\langle R_g^2 \rangle = \frac{1}{N} \sum_{i=1}^{N} (\langle \vec{r}_i - \vec{r}_{cm} \rangle)^2$

$\vec{r}_{cm} = \frac{1}{N} \sum_{i=1}^{N} \vec{r}_i$

$R_g \sim N^{\nu}$

$f(r) = \exp[-U(r)/(k_B T)] - 1$

$v = -\int d^3r \ f(r) = \int d^3r \ (1 - \exp[-U(r)/(k_B T)])$

$D = \frac{1}{2} \lim_{r \to \infty} \frac{(x^2)}{t}$

$D = \frac{k_B T}{\eta_0}$

$L_D = 6\pi \eta a v$

$D = \frac{1}{2} \int_0^\infty dt \ (\vec{v}(t) \cdot \vec{v}(0))$

$\rho(x) = -\epsilon_0 \frac{\partial \psi(x)}{\partial x}$

$\psi(x) \approx \psi_0 e^{-\kappa x}$

$A = k_B T \zeta m$

$J = -A \frac{\partial \zeta}{\partial x}$

$\frac{\partial \zeta}{\partial t} = A \frac{\partial^2 \zeta}{\partial x^2}$

$G(T, S) = G_0 + \frac{1}{2} A(T) S^2 + \frac{1}{3} B S^3 + \frac{1}{4} C S^4$

$U_i(\theta_i) = -A \frac{\partial S}{\partial \theta_i} \left( \frac{3 \cos^2 \theta_i - 1}{2} \right)$
Some vocabulary:
bucket = sango
scoop = kauhoa
fixed = kiinnitetty, pysyy vakiona
everyday = arkinen, tavallinen
solvent = liote, liuotin
order of magnitude = suuruusluokka
contradiction = ristiriita, ristiriitaisuus
resolve = ratkaista, selvittää
phenomenon = ilmiö
term = termi
spherical = pallomainen
extract = erotaa
conditions = ehdot, olosuhteet