TTA-45036 Introduction to Financial Engineering and Derivatives Markets

Exam
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This is a closed-book exam, a non-programmable calculator allowed. You can answer in English or in Finnish. Good luck!

**Question 1.** Explain the following concepts and terms:

a) Efficient markets (1 p)

b) Incomplete markets (1 p)

c) Put option (1 p)

d) Implied volatility (1 p)

e) Short position (1 p)

f) Forward contract (1 p)

**Question 2.**

a) Mathematical finance assumes that financial markets do not allow for profitable arbitrage and that the liquid markets price instruments correctly. Why to use mathematical arbitrage-free models to price options and other derivative securities at all if market prices are already assumed to be correct? (3 p)

b) What is the difference between Black-Scholes implied volatility and historical volatility estimated from time-series? Are you better off using implied volatility or historical volatility to forecast future volatility? Why? (3 p)
Question 3.

a) Show that no-arbitrage bounds for the European put option prices are

\[ P(t,T) < KD(t,T) \]
\[ P(t,T) > (KD(t,T) - S(t))^+ \]

where \( S(t) \) is stock price at time \( t \) and \( P(t,T) \) is the price of a put at time \( t \) with maturity time \( T, T > t \). Moreover, \( D(t,T) \) is a discount factor (with risk-free interest rate) from time \( T \) to time \( t \). Assume that you can borrow or lend any amount of money at the risk-free interest rate. (2 p)

b) Let the current stock price \( S_0 = 1 \), strike price \( K = 0.96 \), continuously compounded interest rates \( r = 0.05 \), time to maturity \( T = 2 \) years, and volatility \( \sigma = 0.25 \). The stock pays no dividends. What is Black-Scholes price of the European call option? (1 p)

c) A stock is worth $10 today and monthly return coefficients are \( U = 1.2 \) and \( D = 1/U \) (i.e. stock price will be either $10 \times 1.2 \) or $10/1.2 after the first month). The continuously compounded risk free interest rate (annual) is 2%. The strike price is $9.5 and time to maturity 2 months. What is the price of a European call option with two-step binomial tree (with \( \Delta t = 1/12 \))? Second, suppose that market price for the European call option is $1. How could one exploit an arbitrage opportunity? (3 p)